

Combinational Logic Design

ONE-MARK QUESTIONS

- 5.1 When a product term cannot be entirely combined with other terms such that a variable will get eliminated, it is called as a prime-implicants (PI) term. There must be pairs of literals for the maximum number of prime implicants. Therefore, for n -variable Boolean function, the maximum number of prime implicants is 2^{n-1} which is option 'd'.
- 5.2 K-map for the sum of products function $f(X, Y, Z) = \sum(2, 3, 4, 5)$ is

XY		00	01	11	10
Z	$\bar{Z}$	0	1	0	1
	Z	0	1	0	1

From the K-map it is clear that the prime implicants are

$\bar{X}Y$ and $X\bar{Y}$ i.e. option 'a'.

- 5.3 We know that the POS is complement of the SOP expression. Therefore, POS representation for the given expression is $F(X, Y, Z) = \prod M(0, 3, 4)$ which is nothing but $(X + Y + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + Y + Z)$ and it is option 'b'.
- 5.4 The given SOP expression is represented as $f(A, B, C) = \sum m(0, 2, 4)$. Therefore, POS representation for the given expression is $F(A, B, C) = \prod M(1, 3, 5, 6, 7)$ which is nothing but $F = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})$ and it is option 'c'.
- 5.5 The given Boolean expression $F(X, Y, Z) = \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ = \sum m(2, 4, 6, 7)$. Therefore, POS representation for the given expression is $F(X, Y, Z) = \prod M(0, 1, 3, 5)$ which is nothing but $F = (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z})$ and it is option 'a'.

TWO-MARKS QUESTIONS

- 5.1 The K-map is

RS\PQ	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$X = 0$

RS\PQ	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

$X = 1$

There are 2 quads in this K-map and hence the simplified SOP expression is $\bar{Q}S\bar{X} + Q\bar{S}X$ and it is option 'b'.

- 5.2 There is one pair and one quad in the given K-map. So, there will be 2 product terms in the minimized sum-of-product expression and it is option 'a'.
- 5.3 Since Boolean expression is given, it is easy to find the essential prime implicants by applying Boolean rules and reducing the given expression.

$$F(w, x, y, z) = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$$

$$\therefore F(w, x, y, z) = wy + xy(1 + \bar{w}z) + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$$

$$\therefore F(w, x, y, z) = y(w + x + \bar{w}\bar{x}) + xz + \bar{x}\bar{y}\bar{z}$$

$$\therefore F(w, x, y, z) = y + xz + \bar{x}\bar{y}\bar{z}$$

$$\therefore F(w, x, y, z) = y + xz + (y + \bar{y})(y + \bar{x}\bar{z})$$

$$\therefore F(w, x, y, z) = y + xz + \bar{x}\bar{z} \quad \text{and it is option 'd'}$$

FIVE-MARKS QUESTIONS

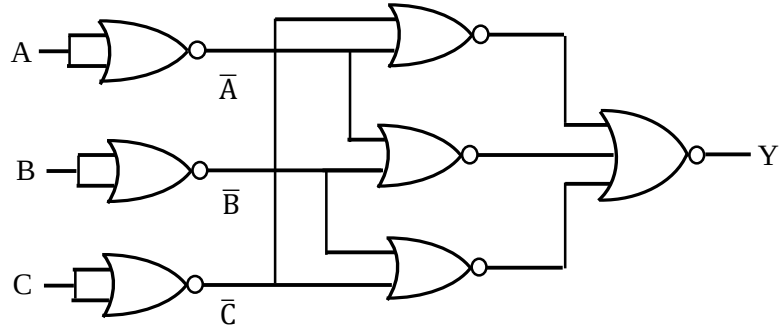
- 5.1 From truth table we can write that $f(C, B, A) = \sum m(0, 1, 2, 5)$. But the logic circuit has to be realized using only NOR gates. It means we need to solve POS expression which is $f(C, B, A) = \sum m(3, 4, 6, 7)$. Therefore, drawing K-map for the given expression

		CB			
		00	01	11	10
$\bar{A}$		1	1	0	0
A		1	0	0	1

From this K-map we can write,

$$Y = (\bar{A} + \bar{C})(\bar{A} + \bar{B})(\bar{B} + \bar{C})$$

Implementation of POS is NOR-NOR logic is as shown below



- 5.2 Karnaugh map for the given function $Y = \sum m(1, 5, 6, 7, 11, 12, 13, 15)$ is

		S_1S_0			
		00	01	11	10
S_3S_2	00	0	1	0	0
	01	0	1	1	1
	11	1	1	1	0
	10	0	0	1	0

SOP expression for the output is

$$Y = \bar{S}_3 \bar{S}_1 S_0 + \bar{S}_3 S_2 S_1 + S_3 S_1 S_0 + S_3 S_2 \bar{S}_1$$

This circuit has to be implemented using four 2-input NAND gates only for the each of the boxes. So, we need to optimize it further.

$$Y = (\bar{S}_3 \bar{S}_1 + S_3 S_1) S_0 + (\bar{S}_3 S_1 + S_3 \bar{S}_1) S_2$$

$$\therefore Y = (\bar{S}_3 \oplus \bar{S}_1) S_0 + (S_3 \oplus S_1) S_2$$

It means digital logic box-1 in the circuit is a XOR function with S_3 and S_1 as inputs. So, the complete logic circuit is as shown below.

